AME40541/60541: Finite Element Methods

Course logistics

AME40541/60541: Finite Element Methods, 3 units
Lecture: MWF 1p-1:50p, B34/36 Geddes Hall
Recitation: Tu 4:30p-5:30p, virtual

This course is being offered as a combined undergraduate (AME40541) and graduate (AME60541) course. The lectures for both listings will be co-located; however, the workload and level of rigor will be higher for those enrolled in AME60541. Undergraduate students can enroll in AME60541 for graduate credit with instructor approval.

Lecture format

All lectures will be delivered in-person from the assigned classroom and broadcast live via Zoom (SP21-AME-40541-01, ID: 920 4573 1035; full Zoom invitation on Sakai). The lectures will be recorded and available on Sakai. A complete Zoom invitation will be emailed to you via Sakai.

Course description

The finite element method is the industry-standard for solving a range of thermal, structural, and fluid flow problems that commonly arise in engineering practice and research. Commercial software is well-suited for solving such problems; however, having a fundamental understanding of the underlying methods is crucial to effectively use the software, develop methods/code tailored to the particular problems they face, or undertake cutting-edge research. This course introduces the fundamental concepts of finite element methods with applications to structural analysis, heat flow, fluid mechanics, and multiphysics problems. It covers the basic topics of linear and nonlinear finite element technology including weak formulations and error analysis, domain discretization on structured and unstructured meshes, assembly of global equations, the isoparametric concept, essential and natural boundary conditions, numerical quadrature, variational crimes, and the structure of a finite element program. Throughout the course, students will build their own finite element code that will be used to investigate fundamental properties of finite element methods. In addition, the course makes use of commercial software to explore more advanced capabilities, validate their own code, and gain experience with software commonly used in engineering industry.

Learning goals

Upon successful completion of this course, you will be able to: (1) identify problems in engineering and science that can (and cannot) be solved with the finite element (FE) method, (2) apply the steps of the FE method to linear/nonlinear partial differential equations (PDEs) and engineering structures, (3) use commercial FE software to solve problems that arise in engineering and science, and (4) develop FE program from scratch that can be used/extended for research purposes.

Prerequisites

MATLAB programming, multivariable calculus, basic linear algebra

Instructor

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Office hours: MW 1:50p-3p, Th 5p-7p, F 10a-12p, 2p-5p (virtual); "open door" policy; or by appointment

Teaching assistants

Tianci Huang
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Note: All office hours (instructor and TAs) will be held virtually using the Zoom meeting for the lectures (SP21-AME-40541-01, ID: 920 4573 1035).

Course material

Website: Sakai
Recommended textbook: Claes Johnson. *Numerical Solution of Partial Differential Equations by the Finite Element Method*. Dover, 2009
Supplementary notes: provided by instructor on the website.

The website for the course will be used to distribute course materials (handouts, slides, code, homework, project), distribute feedback (solutions to homework and the exams), and store grades.

Recitation session

An optional, virtual recitation session will be held every 1-2 weeks. These sessions will be used to dive deeper into material rarely covered in a traditional lecture: background on MATLAB coding, implementational aspects of FEM, interactive office hours, review sessions for examinations, final project assistance, etc.

Finite Element Software: COMSOL Multiphysics

We will use the finite element software COMSOL to explore more advanced properties and features of the finite element method and to validate code you write throughout the course. The School of Engineering has 11 COMSOL licenses installed on machines Bonzai00, Bonzai26-Bonzai35 in Fitzpatrick B19. To access COMSOL, you will need to use one of those machines. I have reserved the room every Thursday 2p-3:30p and Friday 3:30p-5:00p (you Notre Dame ID card is required for entry). To access Fitzpatrick B19 outside of these reserved hours, make sure a class is currently not in session. You can check the schedule posted on the door to plan your visit. COMSOL is also available though the Engineering and Science Computing's (ESC) Virtual Labs (`https://esc.nd.edu/virtual-labs/`), which can be accessed from your personal computer.

Homework

There will be nine homework assignments. They will be a combination of theory, programming, and COMSOL exercises. A total of nine no-questions-asked extension days can be used; however, at most four can be used on a single assignment. Once all extension days are used, a 10% penalty will be applied per day. Submissions more than four days late will not be accepted. All assignments will be submitted via Gradescope.

Examinations

There will be two examinations. The first will be an in-class exam on 3/22 covering the material from chapters 1-4. The second exam will be a take-home exam, assigned 5/10 in-class and due 5/12 in-class, covering the material from chapters 5-10.

Project

There will be a project that will extend the finite element code you develop throughout the semester in the homework assignments to a general finite element code capable of handling unstructured meshes, nonhomogeneous natural boundary conditions, and nonlinear PDEs. You will use the code to solve a number of problems in heat flow, solid mechanics, and fluid mechanics. Checkpoints will be due throughout the second half of the semester. Similar to homework, eight no-questions-asked extension days can be used *per team* (separate from homework extension days); at most four can be used on a single checkpoint.

Grading

The final score will be calculated as a weighted combination of the homework (30%), exams (10% each), and project (50%) scores.

Course schedule

The lecture and assignment schedules below are *tentative* and subject to change.

Lecture	Date	Topics	Reading
1-2	2/3, 2/5	motivation, overview of FEM	PP 1
3-5	2/8, 2/10, 2/12	direct stiffness method	MJZ 1
6-7	2/15, 2/17	mathematical preliminaries	MJZ 2
8-12	2/19-3/1	weighted residual method	MJZ 3
13-16	3/3, 3/5, 3/8, 3/10	FEM, 1d	CJ 1.1-1.3, MJZ 4
17-18	3/15, 3/17	variational formulation	CJ 2, MJZ 5
20	3/22	examination I (Ch 1-4)	-
19, 21-30	3/19-4/16	FEM, higher dimension	CJ 1.4-1.7, 3-5, 12; MJZ 6
31-36	4/19-4/30	FEM, nonlinear	CJ 13, MJZ 7
37-39	5/3, 5/5, 5/7	FEM, time-dependent	CJ 8
40	5/10	FEM, mixed methods	CJ 11
-	5/10-5/12	examination II (Ch 5-10)	-

	Topics	Assigned	Due
HW1	COMSOL	2/3	2/8
HW2	direct stiffness method, COMSOL	2/8	2/15
HW3	direct stiffness method	2/15	2/22
HW4	indicial notation, weak form, COMSOL	2/22	3/1
HW5	weighted residual methods	3/1	3/8
HW6	finite element method	3/8	3/15
Exam 1	in-class, covers Ch 1-4 (HW1-6)	-	3/22
Project	final project	3/17	5/18
HW7	finite element method, COMSOL	3/15	3/24
HW8	finite element method	3/24	4/12
PC1	project checkpoint 1	-	4/12
HW9	sparse matrices, nonlinear equations	4/12	4/19
PC2	project checkpoint 2	-	4/19
PC3	project checkpoint 3	-	4/26
PC4	project checkpoint 4	-	5/3
PC5	project checkpoint 5	-	5/10
PC6	project checkpoint 6	-	5/18
Exam 2	take-home, covers Ch 5-10	5/10	5/12

Sakai

Technical support for Sakai is provided, not by me (instructor) or TAs, but by the OIT Help Desk and the Sakai Team. If you have a question or issue concerning the use of Sakai, please contact the Help Desk at oithelp@nd.edu or phone: 574-631-8111. You can also walk in; they're located in 115 DeBartolo Hall. Support Staff will contact me if they need to discuss the way I've set up our class. Please tell them the web address to our Sakai site, my NetID as your instructor and your NetID, and specifically name the assignment or task you need help with. Technical tips before contacting the OIT Help Desk:

- Access Sakai with a newly opened fresh browser (not one that's been opened for a month and has 47 tabs running across the top).
- Before calling the Help Desk with issues, empty your browser history, close and re-open your browser. Often that's all it takes. (How-To Instructions Here).
- Never login to Sakai twice in two tabs in the same browser, especially when taking a test. You may lose answers.

Policies

Office hours: Students are strongly encouraged to utilize the office hours for assistance with the course material. To make the most out of office hours, students are encouraged to think through the problem on their own and avoid overly generic questions. For example, instead of "I'm stuck on problem 3?", students should explain their thought process, what they have tried, and where they got stuck. If you cannot attend the scheduled office hours, contact the teaching staff directly to schedule an appointment.

Collaboration and honor code: Collaboration is permitted on the homework and project (not exams); however, each student must complete and submit their own assignment. Honor code violations will be handled through appropriate university channels (http://honorcode.nd.edu).

Disabilities: Any student who has a documented disability and is registered with Disability Services should speak with me as soon as possible regarding accommodations. More information can be found at http://disabilityservices.nd.edu.

Privacy statement: Course materials (videos, assignments, problem sets, etc) are for use in this course only. You may not upload them to external sites, share with any person outside of this course, or post them for public commentary without my written permission. We are recording class meetings to support remote students and to provide everyone in the class with useful study aids. These recordings will be available for review through Sakai. The University strictly prohibits anyone from duplicating, downloading, or sharing live class recordings with anyone outside of this course, for any reason.

Health and safety protocols

In this class, as elsewhere on campus, students must comply with all University health and safety protocols, including:

- Face masks that completely cover the nose and mouth will be worn by all students and instructors;
- Physical distancing will be maintained in all instructional spaces;
- Students will sit in assigned seats throughout the semester, which will be documented by faculty for purposes of any needed contact tracing; and
- Protocols for staged entry to and exit from classrooms and instructional spaces will be followed.

We are part of a community of learning in which compassionate care for one another is part of our spiritual and social charter. Consequently, compliance with these protocols is an expectation for everyone enrolled in this course. If a student refuses to comply with the University's health and safety protocols, the student must leave the classroom and will earn an unexcused absence for the class period and any associated assignments/assessments for the day. Persistent deviation from expected health and safety guidelines may be considered a violation of the University's "Standards of Conduct," as articulated in du Lac: A Guide for Student Life, and will be referred accordingly.

Health checks and attendance

Every morning, members of the Notre Dame Community will be asked to complete a daily health check and submit their information via the Return to Campus Advisor application. The health check application will indicate one of the following:

- Student is cleared for class and should attend class in person; or
- Student is advised to stay home to monitor symptoms and should participate in class virtually and complete all assignments and assessments; or
- Student must consult a healthcare provider and should contact University Health Services (UHS) for an assessment. In the meantime, the student should participate in class virtually and complete all assignments and assessments. Depending on the medical assessment, UHS will follow the University's standard protocol for obtaining an excused absence for medical reasons.

Support for student mental health at Notre Dame

Care and Wellness Consultants provide support and resources to students who are experiencing stressful or difficult situations that may be interfering with academic progress. Through Care and Wellness Consultants, students can be referred to The University Counseling Center (for cost-free and confidential psychological and psychiatric services from licensed professionals), University Health Services (which provides primary care, psychiatric services, case management, and a pharmacy), and The McDonald Center for Student Well Being (for problems with sleep, stress, and substance use). Visit `care.nd.edu`.

Additional references

In addition to Johnson's book, there are a number of excellent FEM references, a few of which I have listed below. *These are not required for the course, but are solid addition to one's bookshelf, particularly if continuing with FEM.*

- N.S. Ottosen and H. Petersson. *Introduction to the Finite Element Method*. Prentice-Hall, 1992 (introductory)
- J.N. Reddy. *An Introduction to the Finite Element Method*. McGraw Hill, 3rd edition, 2006 (introductory)
- T.J. Hughes. *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*. Dover, 1st edition, 2000 (introductory, more mathematical)
- Philippe G. Ciarlet. *The Finite Element Method for Elliptic Problems*. Society for Industrial and Applied Mathematics (SIAM), 2002 (advanced, finite element theory)
- Susanne C. Brenner and L. Ridgway Scott. *The Mathematical Theory of Finite Element Methods*. Springer Science & Business Media, 3rd edition, 2008 (advanced, finite element theory)
- Pavel Solin, Karel Segeth, and Ivo Dolezel. *Higher-Order Finite Element Methods*. Chapman & Hall/CRC, 2004 (advanced)
- T. Belytschko, W.K. Liu, and B. Moran. *Nonlinear Finite Elements for Continua and Structures*. John Wiley & Sons, 2013 (advanced)
- J. Hesthaven and T. Warburton. *Nodal Discontinuous Galerkin Methods: Algorithms, Analysis, and Applications*. Springer Science & Business Media, 2007 (introductory, specialized topic)