Lecture 1: Introduction

AME60714: Advanced Numerical Methods

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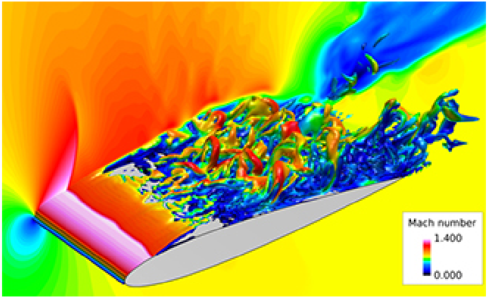
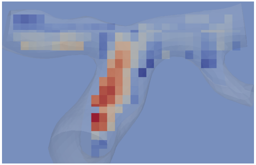
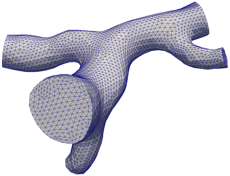
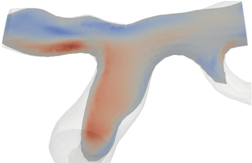
Part I/II: Theory/methods for hyperbolic PDEs

- Hyperbolic PDEs model some of the most complex phenomena encountered in engineering, e.g., gas dynamics and other types of fluid flow
- Numerical methods for (approximately) solving these equations on computers are among the most complicated, rely on intricate mix of approximation theory and the known features of these problems
- Implementation of these methods usually difficult, must scale to supercomputers due to complexity of solutions
- **We will**: 1) study theory of one-dimensional hyperbolic PDEs in details and 2) use this theory to develop finite volume and discontinuous Galerkin methods on general, unstructured grids in d dimensions.



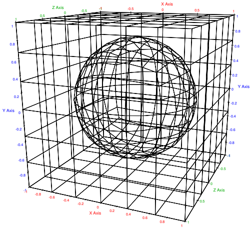
Part III: PDE-constrained optimization

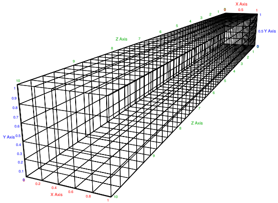
- In science and engineering, often want to do more than simply solve PDEs; want to solve optimization problems involving PDE constraints
- Example in blood flow imaging: want to use computational fluid dynamics to predict patient-specific blow flow from geometry extract from MRI scans; however, boundary and initial conditions unknown → PDE-constrained optimization

- **We will**: 1) develop basic theory behind steady and time-dependent PDE constrained optimization problems, 2) introduce sensitivity and adjoint approaches for the sensitivity and adjoint approaches to compute gradients of optimization functionals involving PDE constraints, and 3) cover basics of numerical optimization.

- Many-query analyses such as optimization and uncertainty quantification require repeated solutions to a primal (and sometimes dual) PDE, which can be extremely expensive for complex problems.
- Projection-based model reduction approximates the state of the PDE using a low-dimensional, data-driven basis; can greatly accelerate PDE solves.
- **Multiscale simulations**: simulation of highly heterogeneous material (composites, biological materials such as bone) can be treated as a multiscale problem where the microstructure is explicitly modeled, but prohibitively expensive; $\mathcal{O}(10^5)$ speedup attained by replacing microstructure with ROM





- **We will**: learn the basics of projection-based model reduction for linear and nonlinear partial differential equations