A Topology Optimization Method with a High-Order Level-Set-Based Boundary Tracking Mesh

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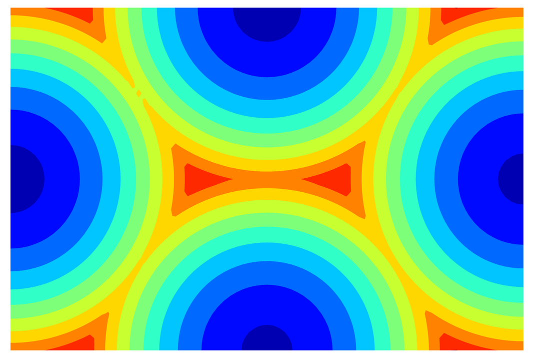
Abstract

We introduce a high-order level-set based boundary tracking topology optimization method. The method allows more accurate representations of the topological structures by moving the mesh nodes to align with features such as holes based on level-set values. The boundaries of the topology, i.e. zero level-sets, are estimated using 2D quadratic functions and second-order isoparametric triangular elements while a robust boundary tracking method is developed to ensure both accuracy and mesh quality. The problem is then solved with a gradient-based optimization solver. In order to promote a unique solution, common techniques such as adding a reasonable penalty to the objective function and smoothing the design variables are also deployed. We present some 2D benchmark examples using our method to demonstrate its validity and efficiency. The method converges faster and offers a smoother representation of boundaries compared to linear methods.



Motivation and Impact

- Topology optimization has been widely studied and applied in numerous engineering applications and is proven to be of great value. In recent years the industrialization of additive manufacturing has enabled vast number of new topological designs with delicate features. The highly digitalized production process gives topology optimization a more prominent role.
- Smooth optimization results that are ready to be fed to the slicing algorithm would be an ideal outcome and calls for more research effort. However, capturing the topology with crisp boundary representation remains a challenge.
- Our method is a step toward better accuracy and efficiency by using higher order elements and aligning the mesh with the topology itself. The optimization process is based upon mathematical programming with consistent gradients throughout to help convergence and speed.

Quadratic boundary tracking method

Governing equations and level-set description

The design domain $\Omega \subset \mathbb{R}^d$ is discretized and represented implicitly by a level-set function. The boundary of the topology is the set $\Gamma = \{\boldsymbol{x} \in \Omega \mid \phi(\boldsymbol{x}) = 0\}$ and the material domain is $\{\boldsymbol{x} \in \Omega \mid \phi(\boldsymbol{x}) > 0\}$ while the void region is $\{\boldsymbol{x} \in \Omega \mid \phi(\boldsymbol{x}) < 0\}$. The problem of finding the optimal topology within the design domain Ω can be restated as finding the level-set function $\phi(\boldsymbol{x})$. We consider a general PDE-constrained topology optimization problem at the fully discrete level

$$\begin{aligned} \underset{\boldsymbol{u},\boldsymbol{\phi}}{\text{minimize}} \quad & f_0(\boldsymbol{u}, \boldsymbol{x}(\boldsymbol{\phi})) \\ \text{subject to} \quad & \boldsymbol{r}(\boldsymbol{u}, \boldsymbol{x}(\boldsymbol{\phi})) = \boldsymbol{0} \\ & l_i \leqslant f_i(\boldsymbol{u}, \boldsymbol{x}(\boldsymbol{\phi})) \leqslant u_i, \quad 1 \leqslant i \leqslant n, \end{aligned} \tag{1}$$

where $\boldsymbol{u}$ is the PDE state variable, $\boldsymbol{\phi}$ is the discrete representation of the level-set function, $\boldsymbol{x}$ contains the nodal coordinates of the mesh, and $\boldsymbol{r}(\boldsymbol{u}, \boldsymbol{x})$ is the discrete PDE. In this work, the mesh nodes $\boldsymbol{x}$ are a function of the level-set values $\boldsymbol{\phi}$ since our method uses the mesh to track the material-void interface for a high-order accurate representation of the geometry and solution. In order to promote unique solutions, we add an extra term to the objective function that drives the level-set toward a signed distance function

$$f_{reg}(\boldsymbol{u}, \boldsymbol{x}(\boldsymbol{\phi})) = \alpha \int_\Omega (|\nabla \boldsymbol{\phi}| - 1)^2 dV, \tag{2}$$

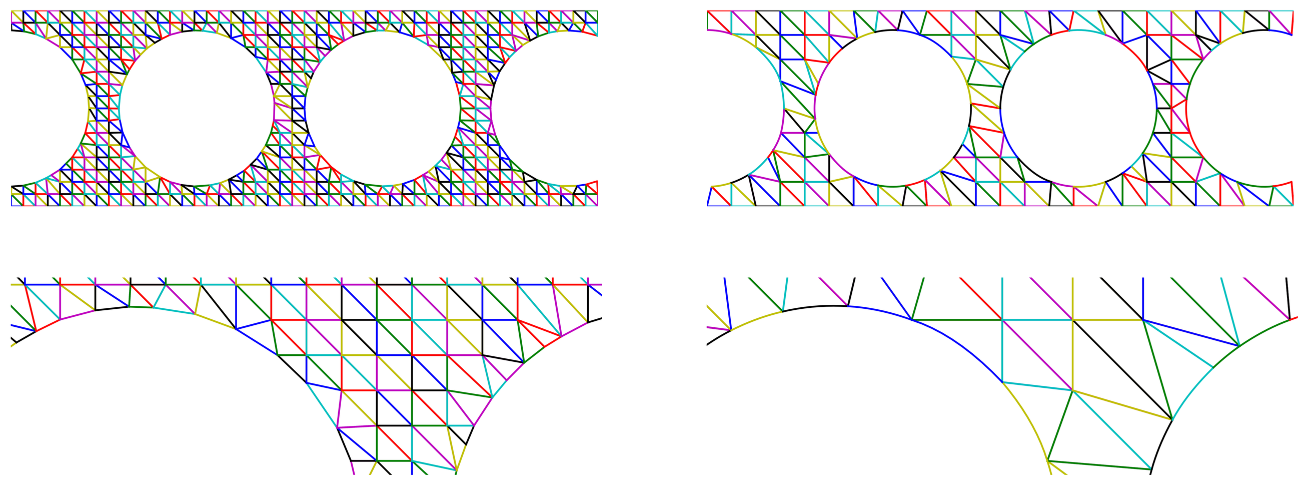
where α is chosen based on the specific problem to balance smoothness and convergence. The sensitivities are then calculated with the reduced space approach

$$\frac{df_i}{d\boldsymbol{\phi}} = \left(\frac{\partial f_i}{\partial \boldsymbol{u}} \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{u}}^{-1} \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{x}} + \frac{\partial f_i}{\partial \boldsymbol{x}} \right) \frac{d\boldsymbol{x}}{d\boldsymbol{\phi}}, \quad 0 \leqslant i \leqslant n. \tag{3}$$

Quadratic boundary tracking algorithm with level-set functions

Boundary tracking with linear elements based on closest-point projection onto the zero level-set has been proposed in the literature [1]. However, we have observed that this approach does not generalize well to higher order elements and proposed an alternate approach. Our method uses quadratic isoparametric triangular elements in 2D as well as quadratic interpolation of the level-set function within an element. First, we identify and move the vertices ignoring the topological features that are too small to be handled effectively to ensure robustness. Next, the middle nodes on edges with moved vertices are moved to positions on the quadratic zero level-set curve where the element can retain good quality. Potential inverted elements are checked and rejected to force middle points of such elements simply stay in the middle of the new vertices. While losing some accuracy this will guarantee the optimization process stable.

Example case to ignore (*left*) where the level-set function approximation intersects at four different points with an element. Example case to avoid (*right*) where reasonable positions at all six nodes generate an inverted element.

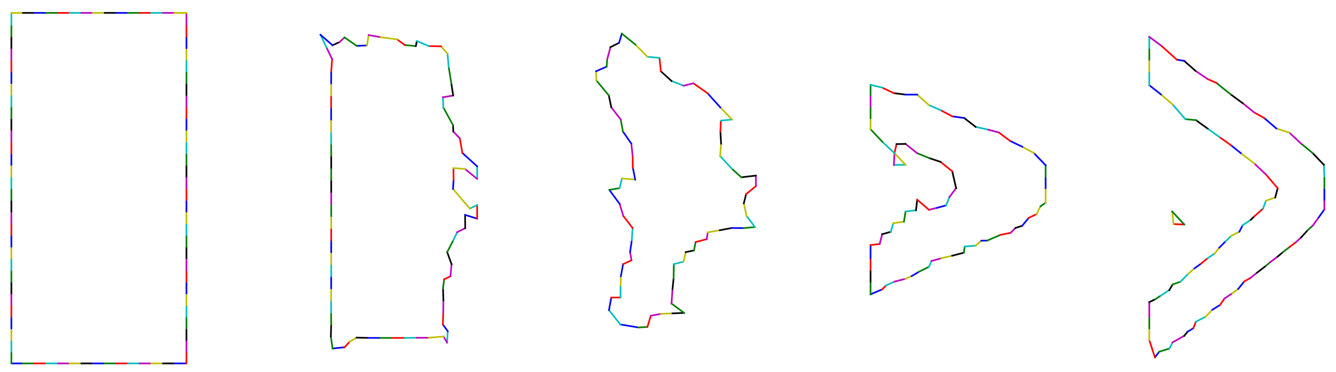


Boundary tracking mesh with linear triangular elements (*left*) and isoparametric quadratic triangular elements, same degrees of freedom. Zoomed-in (*bottom*) figures show the difference in how accurate the boundary is represented.

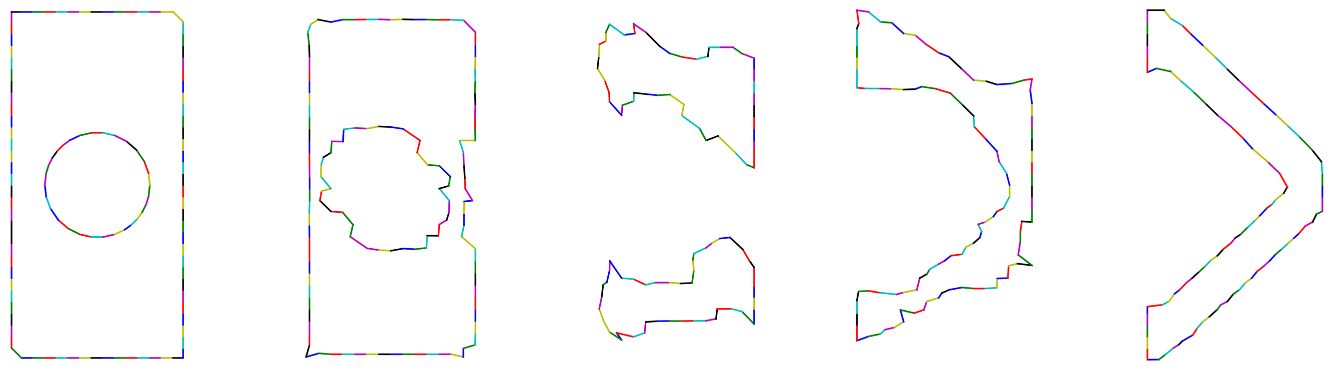
Results: 2D test problems

Here, we solve two benchmark compliance problems in linear elasticity with volume constraints to validate our method. The optimizer we use in this work is IPOPT.

Bar frame compliance problem

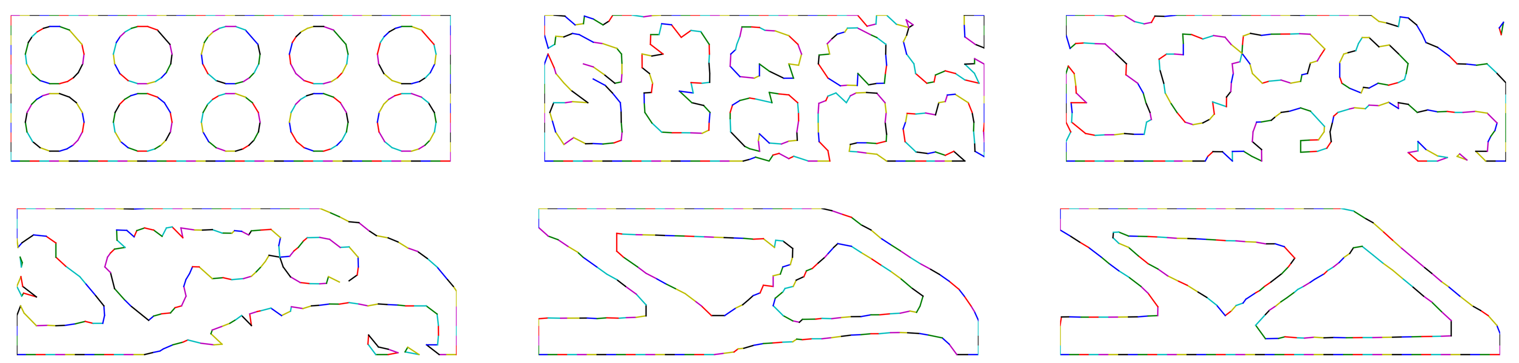


Optimization result of a bar-frame problem with initial topology shown on the *left*. The volume ratio of the final design is constrained at 30%. The left edge of the structure is fixed while a load of 1 unit is applied at the middle of the right edge. Dimension is 2 by 1 unit length. The analytical solution is made of two bars at 45° connected at the right end. α for the regularization objective is set at 0.0001. The mesh is of *15* by *30* elements. *Left to right*: topology at optimization iterations 0, 25, 100, 200, 350.

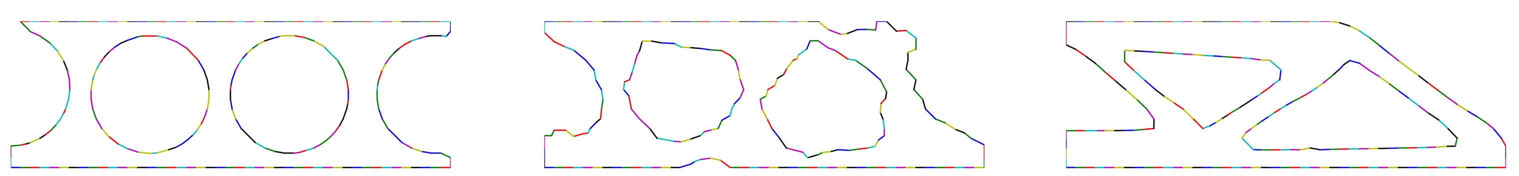


Optimization result of the bar-frame problem with a second initial topology shown on the *left*. Other conditions are the same as above. *Left to right*: topology at optimization iterations 0, 5, 50, 150, 350.

MBB beam compliance problem



Optimization result of the MBB beam problem with initial topology shown at *top-left*. The volume ratio of the final design is constrained at 50%. The left edge of the structure is fixed in the horizontal direction while a load of 1 unit is applied at the top right end. The bottom right corner is also fixed. Dimension is 6 by 2 unit length, with *45* by *15* elements. α for the regularization objective is set at 0.00001. The analytical solution agrees with our result. *Top-left to bottom-right*: topology at optimization iterations 0, 50, 75, 85, 200, 425.



Solution of the MBB beam problem with a second initial topology shown on the *left*. Other conditions are the same as above. *Left to right*: topology at optimization iterations 0, 50, 425.

Acknowledgements

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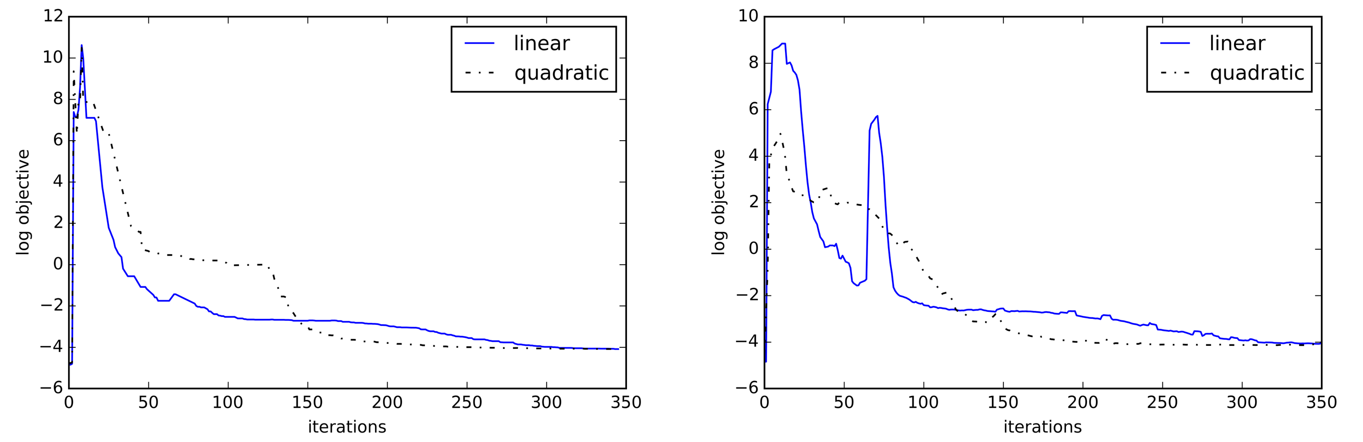
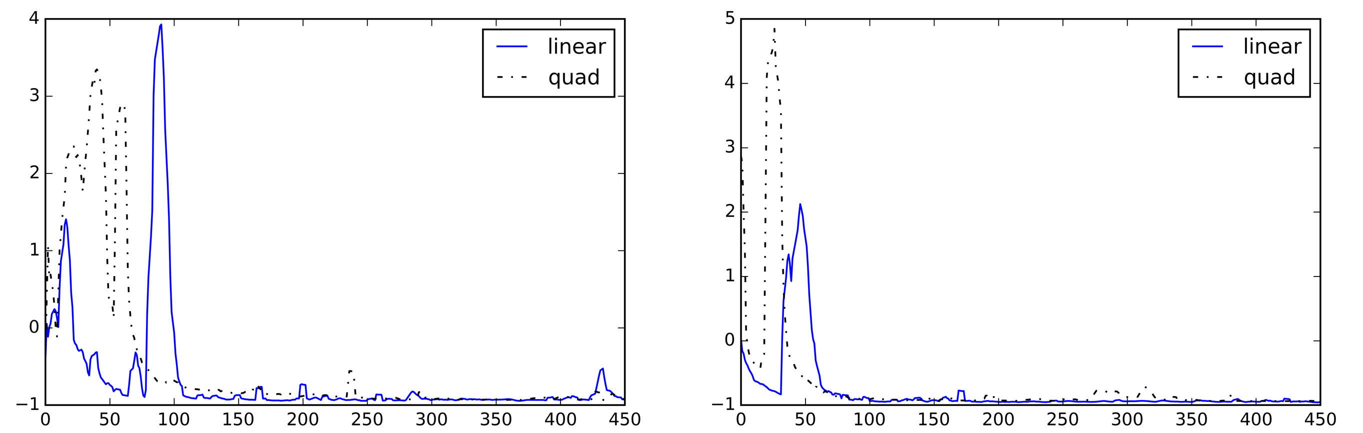
Convergence study of test problems

Finally, we plot the objective function values against the number of iterations for the test problems. We plot together the same problem using both linear and quadratic boundary tracking mesh with the same degrees of freedom to justify the use of higher order elements. The linear method we implemented is similar to the one described in [1]. In all of our examples presented, a Helmholtz density filter was implemented; however, it had little impact on the final solution and is not included in the results shown.



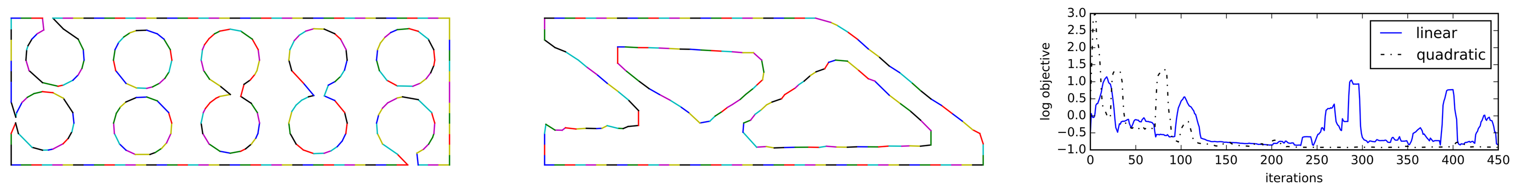
Objective function values vs number of iterations. *Left*: convergence plot of the bar frame problem (first starting point) as described in test problems. *Right*: convergence plot of the bar frame problem (second starting point).



Objective function values vs number of iterations. *Left*: convergence plot of the MBB beam problem (first starting point) as described in test problems. *Right*: convergence plot of the MBB beam problem (second starting point).

As can be seen in the plots, quadratic method shows faster convergence in general. The method with quadratic elements features a more complicated boundary tracking method on a coarser mesh (half the elements of the linear approximation). The difference in computing time between the two approaches on their respective meshes is less than 5%. However, this observation is highly implementation-dependent and further investigation is required to assess the cost of the proposed method.

As a final note, we have observed cases where quadratic method manages to converge while linear method struggles. The linear method particularly struggles when the mesh is coarse, which further showcases the potential for using higher order elements.



Case where quadratic method converges and linear fails with same total degrees of freedom of 2465 and same α. Number of elements is *42* by *14* for quadratic case. *left*: Initial topology. *middle*: Topology using quadratic tracking at iteration 450. *right*: Objective vs iterations under this condition. Linear case doesn't work well with optimizer.

Conclusions and Future work

- A 2D quadratic boundary tracking topology optimization method successfully implemented and tested.
- The method provides good representation of the boundaries and can reduce the number of iterations compared to linear tracking method in many cases.
- Future work will focus on expanding the set of initial conditions that will converge by being able to generate voids and expanding the method to 3D and even higher order.

References

[1] Shintaro Yamasaki, Atsushi Kawamoto, Tsuyoshi Nomura, and Kikuo Fujita. A consistent grayscale-free topology optimization method using the level-set method and zero-level boundary tracking mesh. *International Journal for Numerical Methods in Engineering*, 101(10):744–773, 2015.